From real world to exponential function

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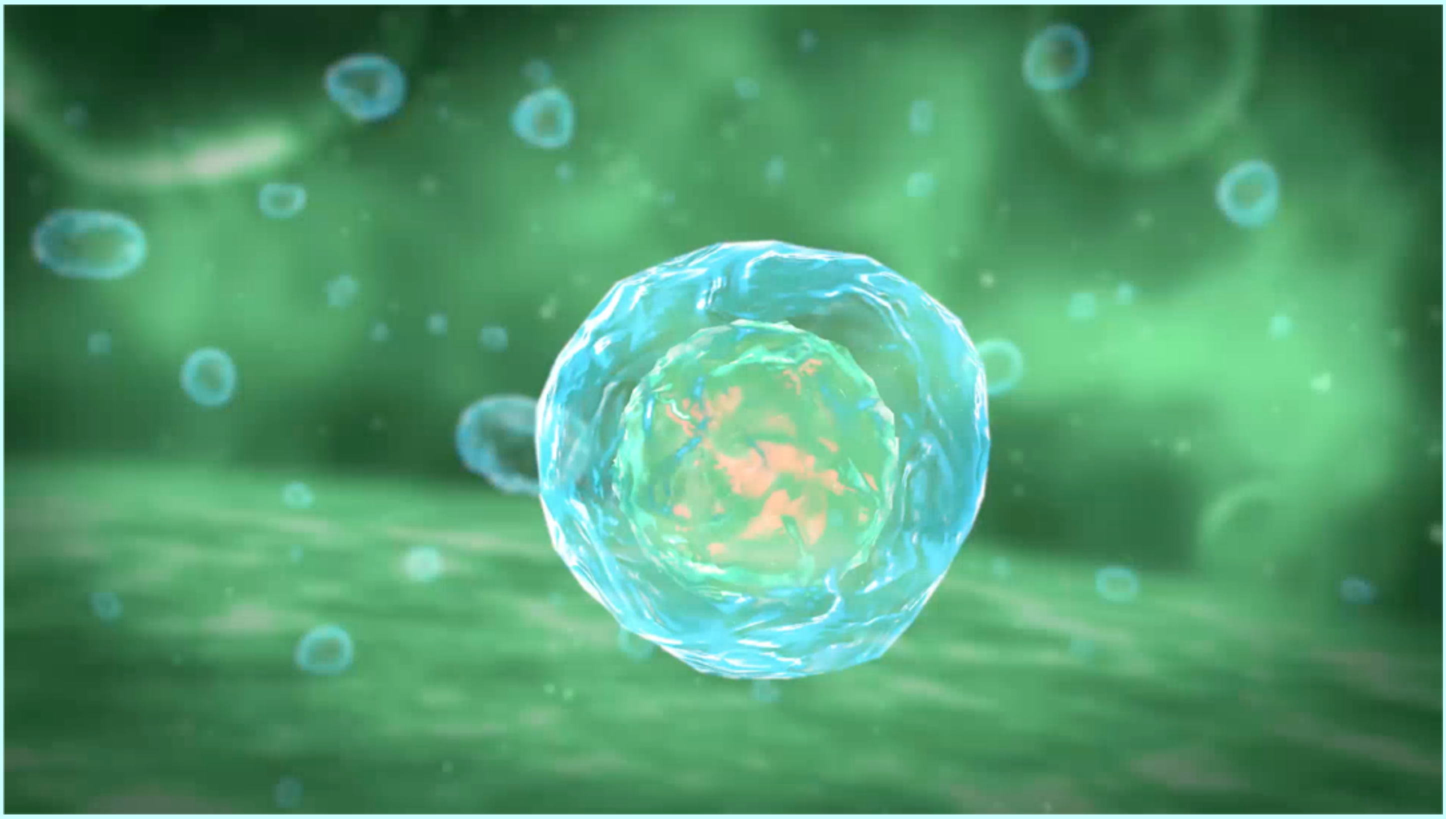
Observe two natural phenomena through ‘mathematical eyes’

- **Bacterial growth through binary fission.**
- **Radioactive decay of radiocarbon**.

Bacterial reproduction



To reproduce, a bacterium repeatedly splits into two, as shown in the video

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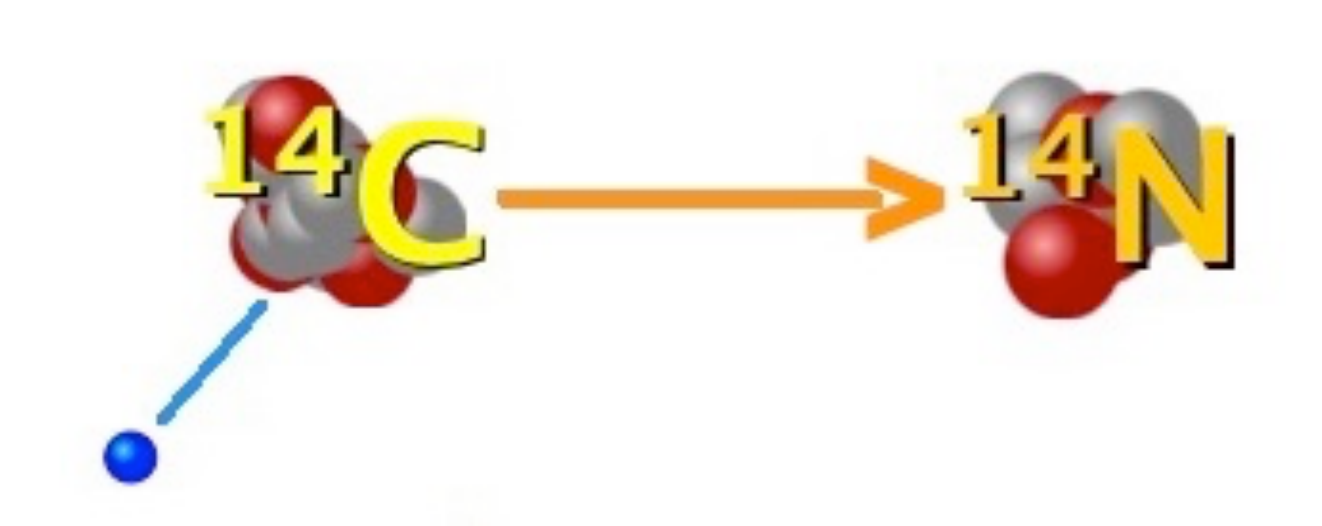
Radioactive decay of radiocarbon



Carbon-14 (^{14}C) is a radioactive isotope of carbon that occurs naturally.

^{14}C gradually decays, transforming into nitrogen-14 (^{14}N).

Approximately half of the ^{14}C atoms decay into ^{14}N in about 6,000 years (half-life).

The following video illustrates a simulation of this process

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Radioactive decay of radiocarbon

Simulation

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Activity.
From real world to exponential law

Complete the worksheet to analyze the two processes through ‘mathematical eyes’.

What did you find?

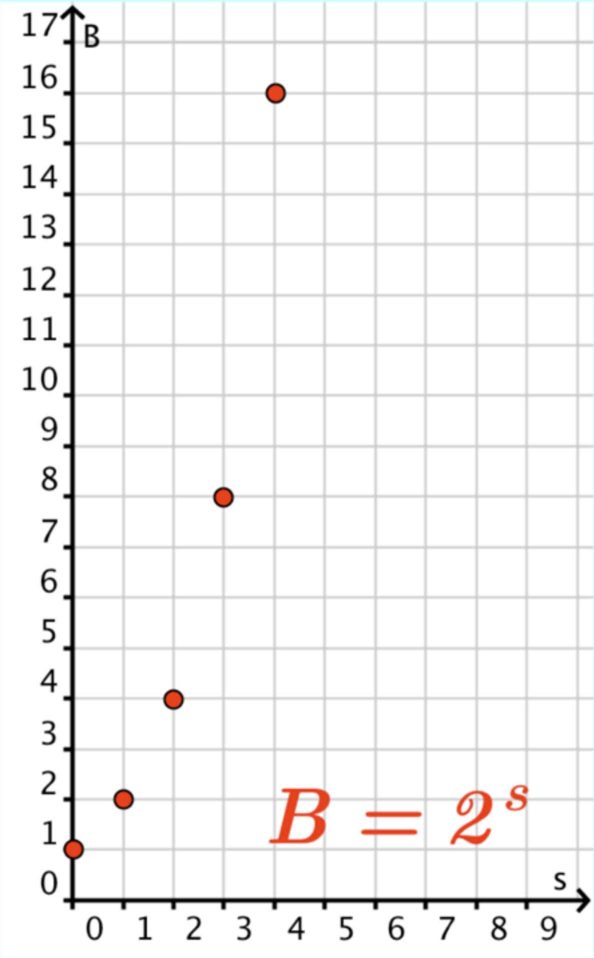
1. Mathematical law describing bacterial growth
2. Mathematical law describing radioactive decay

Let's review some key stages of the path you have followed

1. Bacterial reproduction law

Number of splits *s*	Number of bacteria *B*
0	1
1	2
2	$4 = 2^2$
3	$8 = 2^3$
10	2^{10}

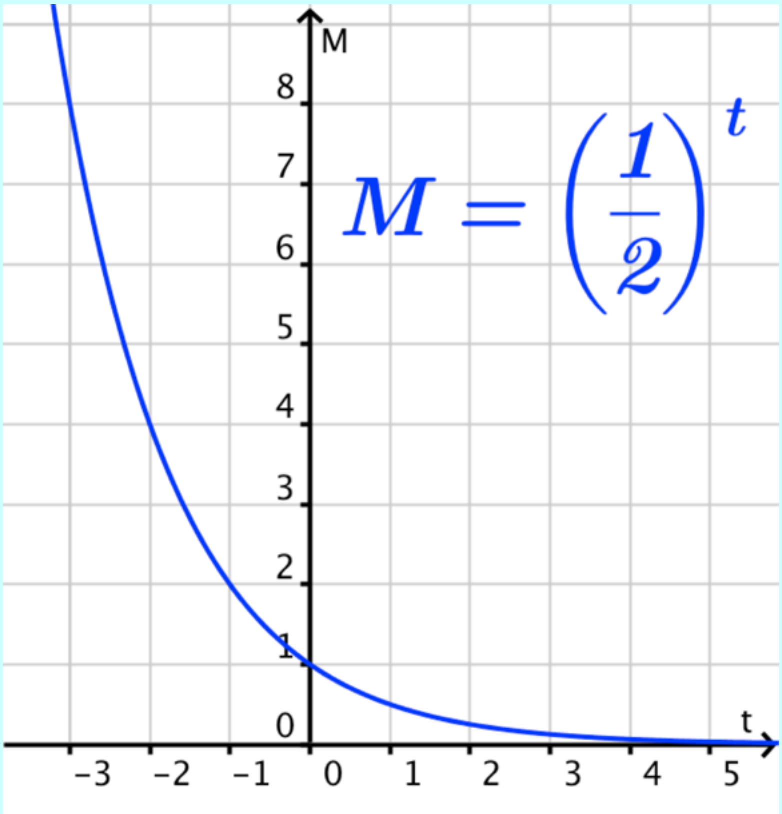
$$B = 2^s$$



I cannot think of −2 splits or 4.5 bacteria.
I can replace *s* only with any natural number.
The values of B are natural numbers (excluding 0)

2. Radioactive decay law

Time t	C_{14} Mass M
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4} = \left(\frac{1}{2}\right)^2$
3	$\frac{1}{8} = \left(\frac{1}{2}\right)^3$
10	$\left(\frac{1}{2}\right)^{10}$



$$\boxed{M = \left(\frac{1}{2}\right)^t}$$

I can think of the past as ‘negative time’, but I cannot have a negative mass of carbon-14

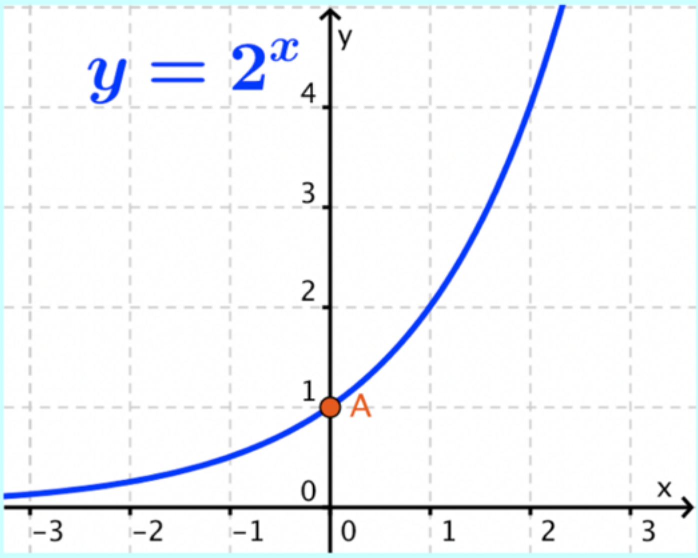
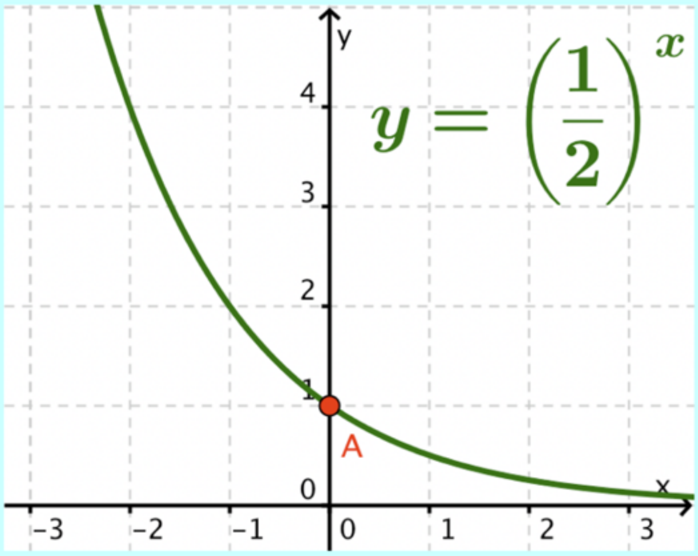
The exponential function

These natural phenomena help explain why the *exponential function* appears in mathematics.

Exponential function

$$y = b^x$$

Where *b* is a positive real number, *x* is a real variable, y is a real positive variable.

$b = 2$	$b = \frac{1}{2}$
	

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